In 2018, around 8% of the world energy was generated from renewable sources. Hydroelectricity is the main source of renewable energy, with 52% of these 8%. However, energy production is concentrated on large potential sites and mostly excludes small and micro hydroelectricity. For instance, in Europe, the number of sites compatible with micro hydro plants (<100kW) is estimated at 350 000 and represents around 5GW.

Small scale hydraulic power has many advantages such as local economic development and autonomy, low floor space requirements, and easily predictable electricity production.

Many prime movers (turbines or water wheels) have been developed to extract hydraulic power in various operating conditions. Kaplan, Pelton and Francis turbines represent most of the power installed through the world, since they are adapted to large head and flow rates, and thus high power. However, the efficiency of these turbines for low head and flow rate is really poor.

Stream water wheels only use the kinetic energy, require high flow rates and have poor efficiency (<40 %).

Gravity machines such as Archimedes screws and hydrostatic pressure machines (HPMs) use the pressure force of the water weight acting on their blades. If Archimedes screws can reach an efficiency between 70% and 90%, large civil engineering is required and limits their use to relatively large sites, where a high initial investment is acceptable.

In this context, HPMs combine both good efficiency (up to 70 %), limited environmental impact and low initial investments.
Principle of a hydrostatic water wheel

Low head water wheels, usually called low breastshot or undershot water wheels, have existed for several centuries under different forms. The two main competing models are the Sagebien and Zuppinger water wheels, depicted in Figure 2.

![Figure 2: The two main historic models of low breastshot gravity wheels](image)

The Sagebien water wheel is designed to minimize inflow power losses, when the Zuppinger water wheel is designed to minimize exit power losses.

The hydropower converter developed at PYTHEAS Technology is based on an innovative water wheel first studied during a thesis carried out at Southampton University. It is based on the principle of hydrostatic pressure machines and is depicted in Figure 3. With its curved blades, it combines the advantages of the Sagebien and the Zuppinger water wheels.

![Figure 3: Schematic of the hydrostatic pressure machine developed at PYTHEAS Technology](image)

It acts as a weir and holds water upstream of the converter. It consists of two distinct parts: a central hub (diameter H), which corresponds to the water head difference, and blades, which have a length equal to the downstream water depth. The number of blades is fixed here at twelve, regularly spaced around the hub.
The operation of a HPM can be decomposed into four principle phases, based on the position of the blades considered, as shown in Figure 4.

**Induction**: it includes the moment when the blade dives into the water and the period of cell filling. At the end of this stage, the water is moving at the same speed as the wheel.

**Compression**: at the end of this stage, the water contained in the cell is no longer in contact with the upstream current. During this stage, no mass is added to the cell. However, the pressure keeps increasing.

**Power**: it is during this stage that energy is transferred from the fluid to the converter.

**Exhaust**: at this point, pressure drops quickly and the fluid is released downstream. This stage also contains the emptying of the cell.

Most of the losses appear during the **induction** and **exhaust** stages. The blade penetrating the water interface causes large flow disturbances, which are significant in terms of power losses.

Moreover, some **two-phase mixing** (air entrained by water) can be observed during these periods and interferes with a smooth running system.

The **compression phase is also critical to ensure proper operation of such water wheels**. The pressure has to be at its maximum at the end of this stage to provide a high efficiency during the power stage.

![Figure 4: Cycles describing operation of a hydrostatic pressure machine](image)
Analytic model

The upstream water generates a pressure $p_1$ on the blade (A) and the central hub (B), whereas $p_2$ is applied by the downstream water on the blade (C). It gives us the following total pressure force on the wheel:

$$F_p = \rho g B (h_1 - h_2 - \Delta h)$$

And the ideal power produced by such a device can be written as:

$$P_{\text{ideal}} = \rho g h_2 v_2 \left( h_1 - h_2 - \frac{v_2^2}{2g} \left( 1 - \left( \frac{h_2}{h_1} \right)^2 \right) \right)$$

It can be noticed that this power calculation already takes into account part of the losses due to the acceleration of the fluid at the entrance of the water wheel (with the term $\Delta h$).

To extract a maximum amount of energy during the powering stage (3rd step), the **floor's shape must be adapted**. During the compression stage, no mass exchange should occur between the cell and the external flow, which is impossible if the floor under the device is flat. **Dissipation** would occur during phase 2 because energy is returned to the external flow.

One way of solving this issue is to use a shroud. This shell is placed on the floor and follows the blades’ trajectory. No energy is wasted during phases 2 and 3.
Computational Fluid Dynamics

The analytic model is based on assumptions which are usually not satisfied.

For instance, losses created by turbulences in the flow must be taken into account. Moreover, it assumes that the blade is straight in the width, while in fact, the blade is twisted to ease the filling and emptying of the cells.

To obtain a more accurate estimation of the power produced, computational fluid dynamics (CFD) is performed, where the Navier-Stokes equations are solved numerically.

The flume must be wider than the converter to avoid capturing air or water during the transition phases (filling and draining). Therefore, to hold the wheel, a bottleneck is required. It's materialized by the two side walls.

Initial Conditions

For the simulations, the rotation velocity (ω in rad.s⁻¹) of the wheel is kept constant. The flow rate through the device is directly proportional to its rotation velocity. However, it should be remembered that neither the velocity nor the torque will be constant during power generation. The system will find its own balance in function of the upstream water height and these parameters will vary as a function of time.

A no-slip condition is applied on every wall in the system and the mesh is refined close to the walls to respect the « y+ condition » for turbulence model's hypothesis.

To save computational time, water levels are initialized as follows: upstream, water reaches the top of the central hub and downstream, it is tangential to the bottom of the hub (see Figure 9).
Results and comparison

As a reminder, the mean output power and the total efficiency are calculated as follows:

\[ P_{\text{out}} = \omega T \]
\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\omega T}{\varrho g H Q} \]

The mean power produced simulated by CFD (in red) is compared to the one given by the analytical model in Figure 10. It shows a good correlation between the two methods, especially at low speed. The gap between the two datasets is below 2% for the rotation velocities below 20 rpm. This difference rises up to 6.6% for velocities above 20 rpm, but remains relatively low. This deviation between the two models could be explained by the underestimation of the turbulence losses at high velocity in the analytical model. In this model, turbulence is only represented by a force, which depends on an empirical coefficient.

The peak of power and the form of the curve around this point are the major deviations observed here. CFD’s power seems to reach a plateau whereas the analytical model seems to produce a parabolic curve. Moreover, the maximum power is not obtained at the same rotational speed. To reach the maximum power, the wheel must spin 30% faster with the analytical model. It can be explained by the blade’s analytical geometry which does not model correctly energy exchanges during the cell filling and emptying phases.

The plot also shows analytical and CFD efficiencies (in blue, right scale). Both of the models calculate the efficiency with the equation above. Efficiency is large (~80%) for low rotational speed. However, power values are low for these speeds and do not exceed 7W, which explains why CFD was not use for this range of rotational speed.

Efficiency curves have similar trends with both datasets. However, efficiencies obtained by CFD are significantly lower. The deviation between the two datasets remains constant, around 15%.

The power calculation has only one variable (the torque), which allows great agreement between CFD and analytic model. The efficiency calculation combines uncertainties on the head, flow rate and torque, leading to higher deviation between the two methods.

In CFD, which should better model the reality, for this scale model and these rotational speeds, efficiencies are between 50% and 66%. As expected, power reaches its maximum for higher speeds than the efficiency. The device produces a maximum of 16W for a speed of 25 rpm and an efficiency of 54%. Moreover, it should be noticed that the efficiency, from CFD data, seems to decrease linearly with the rotational speed, which confirms observations made at Darmstadt University.

Figure 10: Comparison between CFD and analytic model
Advantages of hydrostatic pressure machines

This converter presents many advantages compared to the existing prime-mover (Pelton, Kaplan, Francis, Archimedes screw...).

At the opposite of most commercial devices, this system operates at both **low head and low flow rate**. These conditions are often observed on installation sites such as mills, small weirs or irrigation flumes.

Compared to current turbines, it has a **good efficiency at these challenging operating conditions**.

The system is designed to be a **turnkey solution**, with very low civil engineering requirements.

Due to the linear relationship between flow rate and the wheel's rotational speed, the **wheel can easily act as a control system** such as a weir or a sluice gate. The energy dissipated in these regulation tools is a great source of power and should not be lost when controlling head or discharge.

As hydrostatic pressure machines act **naturally as a weir**, variations in terms of upstream head are almost suppressed and allow a constant water level.

Analytic, CFD and experimental studies have brought a good understanding of this turbines operation, allowing to adapt the design of the wheel or the shape of the shroud to **optimise the power produced in function of the chosen installation site**.

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**Nomenclature**

- $\rho$: Fluid density
- $g$: Gravity acceleration
- $v$: Fluid velocity
- $H$: Head difference across the water wheel
- $h$: Water level
- $B_i$: Blade width
- $p$: Pressure
- $Q$: Volumetric flow rate
- $T$: Torque
- $P$: Power
- $\omega$: Rotational speed
- $\eta$: Efficiency